*Journal of Thermal Analysis, Vol. 37 (1991) 2729-2730*

*Note*

# ON JOHNSON-MEHL-AVRAMI EQUATION
## A comment on Sarkar and Ray's criticism about the application of JMA equation to the analysis of kinetic data

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(Received April 14, 1991)

The contention of Sarkar and Ray that Johnson-Mehl-Avrami equation is not a true kinetic equation is disproved. It is shown that this equation can be expressed in the form of a standard kinetic equation. The results of the authors are only an approximation of Johnson-Mehl-Avrami kinetics when the fraction transformed is small.

Johnson-Mehl-Avrami equation [1, 2]

$$1-\alpha=\exp-(K(T)t)^n \tag{1}$$

where $\alpha$ is the fraction transformed till time $t$, $K$, $(T)$ is the rate constant and $n$ the growth exponent had been widely used to study the kinetics of transformation involving nucleation and growth [3–5]. In a recent paper [6] Sarkar and Ray have suggested that this equation is not a true kinetic equation on the premise that it cannot be expressed in the form of a standard kinetic equation

$$d\alpha/dt=K(T)f(\alpha) \tag{2}$$

where $K(T)$ is the rate constant and $f(\alpha)$ is a function of $\alpha$. This is not so since Eq. (1) after suitable manipulation can be expressed in a form given by Eq. (2).

Differentiation of Eq. (1) yields

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$$\mathrm{d}\alpha/\mathrm{d}t = nK^n(T)(1-\alpha)t^{n-1} \tag{3}$$

Again from Eq. (1) we get

$$t = \{\ln[1/(1-\alpha)]\}^{1/n}/K(T) \tag{4}$$

Substituting in Eq. (3) for $t$ given by Eq. (4)

$$\mathrm{d}\,\alpha/\mathrm{d}t = nK(T)(1-\alpha)\{\ln[1/(1-\alpha)]\}^{(n-1)/n} \tag{5}$$

Equation (5) is the same as the standard kinetic equation given by Eq. (2) with

$$f(\alpha) = n(1-\alpha)\{\ln[1/(1-\alpha)]\}^{(n-1)/n} \tag{6}$$

For small values of $\alpha$ Eq. (5) becomes

$$\mathrm{d}\,\alpha/\mathrm{d}t = nK(T)(1-\alpha)\alpha^{(n-1)/n} \tag{7}$$

which, according to the authors, is the modified true kinetic equation.

Taking logarithm of Eq. (5) we get

$$\ln(\mathrm{d}\,\alpha/\mathrm{d}t) = \ln(nK(T)) + \ln(1-\alpha) + [(n-1)/n]\ln\{[1/((1-\alpha)]\} \tag{8}$$

$$\mathrm{d}\ln(\mathrm{d}\,\alpha/\mathrm{d}t)/\mathrm{d}\ln\alpha = \mathrm{d}\ln(nK(T))/\mathrm{d}\ln\alpha - \alpha/(1-\alpha) + + (n-1)/n\{\alpha/[(1-\alpha)\ln(1-\alpha)]\} \tag{9}$$

when $\alpha<1$ Eq. (9) reduces to

$$\mathrm{d}\ln(\mathrm{d}\,\alpha/\mathrm{d}t)/\mathrm{d}\ln\alpha = \mathrm{d}\ln(nK(T))/\mathrm{d}\ln\alpha - \alpha/(1-\alpha) + (n-1)/n \tag{10}$$

This is the equation (Eq. (15) of [6]) the authors used for kinetic plots in their paper. In the plot of log(d $\alpha$/d$t$) *vs*. log $\alpha$ (Fig. 7 of [6]) the authors claim that the gradient is given by $-\alpha/(1-\alpha)$, a term that increases monotonously. If this were the fact the curves would have ever increasing slopes. On the contrary, their plots have slopes that initially increase, reach a maximum and then decreases again, a fact that cannot be accounted by Eq. (10) but can be explained on the basis of Eq. (9).

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